

STUDENT #: \_\_\_\_\_

NAME: \_\_\_\_\_

## ASSIGNMENT 7: Wave Optics

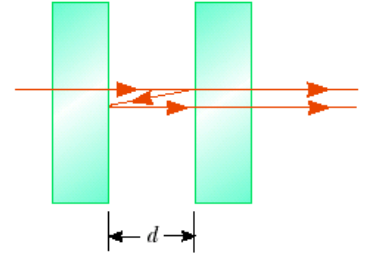
Released: March 3

Due: March 10

6PM

(32 points total)

1. A beam of 580-nm light passes through two closely spaced glass plates, as shown in Figure . For what minimum nonzero value of the plate separation  $d$  is the transmitted light bright?



If the path length difference  $\Delta = \lambda$ , the transmitted light will be bright.

Since  $\Delta = 2d = \lambda$ ,

$$d_{\min} = \frac{\lambda}{2} = \frac{580 \text{ nm}}{2} = \boxed{290 \text{ nm}}.$$

2. A possible means for making an airplane invisible to radar is to coat the plane with an antireflective polymer. If radar waves have a wavelength of 3.00 cm and the index of refraction of the polymer is  $n = 1.50$ , how thick would you make the coating?

Treating the anti-reflectance coating like a camera-lens coating,  $2t = \left(m + \frac{1}{2}\right) \frac{\lambda}{n}$ .

$$\text{Let } m = 0: \quad t = \frac{\lambda}{4n} = \frac{3.00 \text{ cm}}{4(1.50)} = \boxed{0.500 \text{ cm}}.$$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar—to 1.50 cm—now creating maximum reflection

3. When a liquid is introduced into the air space between the lens and the plate in a Newton's-rings apparatus, the diameter of the tenth ring changes from 1.50 to 1.31 cm. Find the index of refraction of the liquid.

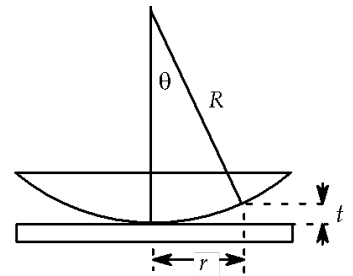
The condition for bright fringes is  $2t + \frac{\lambda}{2n} = m \frac{\lambda}{n}$   $m = 1, 2, 3, \dots$

From the sketch, observe that  $t = R(1 - \cos \theta) \approx R \left(1 - 1 + \frac{\theta^2}{2}\right) = \frac{R}{2} \left(\frac{r}{R}\right)^2 = \frac{r^2}{2R}$ .

The condition for a bright fringe becomes  $\frac{r^2}{R} = \left(m - \frac{1}{2}\right) \frac{\lambda}{n}$ .

Thus, for fixed  $m$  and  $\lambda$ ,  $nr^2 = \text{constant}$

Therefore,  $n_{\text{liquid}} r_f^2 = n_{\text{air}} r_i^2$  and  $n_{\text{liquid}} = (1.00) \frac{(1.50 \text{ cm})^2}{(1.31 \text{ cm})^2} = \boxed{1.31}$ .



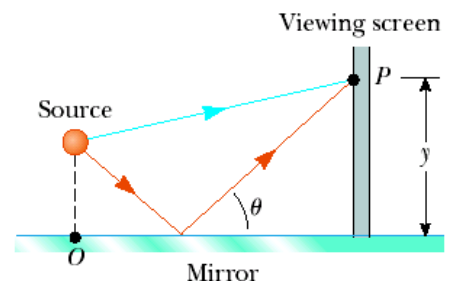
4. Interference effects are produced at point  $P$  on a screen as a result of direct rays from a 500-nm source and reflected rays from the mirror, as shown. Assume the source is 100 m to the left of the screen and 1.00 cm above the mirror. Find the distance  $y$  to the first dark band above the mirror.

$$x = 1.22 \frac{\lambda}{d} D = 1.22 \left( \frac{5.00 \times 10^{-7} \text{ m}}{5.00 \times 10^{-3} \text{ m}} \right) (250 \times 10^3 \text{ m}) = \boxed{30.5 \text{ m}}$$

$$D = 250 \times 10^3 \text{ m}$$

$$\lambda = 5.00 \times 10^{-7} \text{ m}$$

$$d = 5.00 \times 10^{-3} \text{ m}$$



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A) A binary star system in the constellation Orion has an angular interstellar separation of  $1.00 \times 10^{-5}$  rad. If  $\lambda = 500$  nm, what is the smallest diameter the telescope can have to just resolve the two stars?

$$D = 1.22 \frac{\lambda}{\theta_{\min}} = \frac{1.22(5.00 \times 10^{-7})}{1.00 \times 10^{-5}} \text{ m} = \boxed{6.10 \text{ cm}}$$

B) How far above the horizon is the Moon when its image reflected in calm water is completely polarized?

( $n_{\text{water}} = 1.33$ )

Complete polarization occurs at Brewster's angle  $\tan \theta_p = 1.33$  so that  $\theta_p = 53.1^\circ$ .

Thus, the Moon is  $\boxed{36.9^\circ}$  above the horizon.

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A) Unpolarized light passes through two polaroid sheets. The axis of the first is vertical, and that of the second is at  $30.0^\circ$  to the vertical. What fraction of the incident light is transmitted?

The average value of the cosine-squared function is one-half, so the first polarizer transmits  $\frac{1}{2}$  the light. The second transmits

$$\cos^2 30.0^\circ = \frac{3}{4}. \quad I_f = \frac{1}{2} \times \frac{3}{4} I_i = \boxed{\frac{3}{8} I_i}$$

B) One arm of a Michelson interferometer contains an evacuated cylinder of length  $L$ , having glass plates on each end. A gas is slowly leaked into the cylinder until a pressure of 1 atm is reached. If  $N$  bright fringes pass on the screen when light of wavelength  $\lambda$  is used, what is the index of refraction of the gas?

Counting light going both directions, the number of wavelengths originally in the cylinder is  $m_1 = \frac{2L}{\lambda}$ . It changes

to  $m_2 = \frac{2L}{\lambda/n} = \frac{2nL}{\lambda}$  as the cylinder is filled with gas. If  $N$  is the number of bright fringes passing,

$$N = m_2 - m_1 = \frac{2L}{\lambda}(n - 1), \text{ or the index of refraction of the gas is } n = \boxed{1 + \frac{N\lambda}{2L}}$$

7 Many cells are transparent and colorless. Structures of great interest in biology and medicine can be practically invisible to ordinary microscopy. An *interference microscope* reveals a difference in refractive index as a shift in interference fringes, to indicate the size and shape of cell structures. The idea is exemplified in the following problem: An air wedge is formed between two glass plates in contact along one edge and slightly separated at the opposite edge. When the plates are illuminated with monochromatic light from above, the reflected light has 85 dark fringes. Calculate the number of dark fringes that appear if water ( $n = 1.33$ ) replaces the air between the plates.

For dark fringes,

$$2nt = m\lambda$$

and at the edge of the wedge,

$$t = \frac{84(500 \text{ nm})}{2}$$

$$2nt = m\lambda$$

When submerged in water,

$$m = \frac{2(1.33)(42)(500 \text{ nm})}{500 \text{ nm}}$$

$$\text{so } m + 1 = \boxed{113 \text{ dark fringes}}$$

